HYDRODYNAMICS OF A LIQUID WITH DEFORMABLE AND INTERACTING PARTICLES

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Interest in the hydrodynamics of a liquid with particle rotations and microdeformations has recently intensified [1-9] in connection with the technical applications of different artificially synthesized structured media. A model of a liquid with deformable microstructure was first proposed in [4] and was thermodynamically analyzed in [6], in which a model of a liquid was constructed by means of methods from the thermodynamics of irreversible processes. A model of a macro- and microincompressible liquid with particle rotations and deformations has been proposed [7, 8] based on constitutive equations from [6]. Below we will solve the sphere rotation problem in an infinite liquid given different boundary conditions on the rates of particle rotation and microdeformation within the context of the system of equations presented in [7]. The solution of an analogous problem for a micropolar liquid simulating a suspension with solid particles has been obtained [9] and the solution for a viscous liquid was found by Stokes in [10].

§1. The motion equations for a macro- and microincompressible isothermic liquid can be symbolically represented in the form [7]

$$\rho(\mathbf{f}-\mathbf{v}^{\cdot})-(\alpha_7+\alpha_4)\nabla\times\nabla\times\mathbf{v}+2\alpha_4\nabla\times\boldsymbol{\omega}-2(\alpha_7-\alpha_8)\nabla\cdot\mathbf{v}-\nabla p=0; \tag{1.1}$$

$$\rho(2\mathbf{l}-\mathbf{b})+2\alpha_4(\nabla\times\mathbf{v}-2\boldsymbol{\omega})+\left(\gamma_1+\frac{2}{3}\gamma_{10}\right)\nabla\nabla\cdot\boldsymbol{\omega}-\left(\gamma_3+\frac{1}{2}\gamma_{10}\right)\nabla\times\nabla\times\boldsymbol{\omega}+2\left(\frac{1}{8}\gamma_{11}-a_{11}\right)\nabla\times\nabla\cdot\boldsymbol{\nu}=0;$$

$$\rho(\mathbf{B}^s-\mathbf{b}^s)-\mathbf{P}^s+2(\alpha_7-\alpha_8)(\nabla\mathbf{v})^s-d\boldsymbol{\nu}-(1/3\gamma_{13}+1/4\eta_{11})$$
$$\times(\nabla\times\nabla\times\boldsymbol{\nu})^s+2\left(a_{10}+\frac{11}{30}\gamma_{13}-\frac{1}{16}\eta_{11}\right)(\nabla\nabla\cdot\boldsymbol{\nu})^s+2(a_{11}$$
$$-1/8\gamma_{11})[\nabla(\nabla\times\boldsymbol{\omega})]^s-\left(\frac{2}{3}a_{10}+\frac{2}{15}\gamma_{13}-\frac{1}{8}\eta_{11}\right)(\nabla\nabla\cdot\mathbf{v})\,\mathbf{U}$$
$$+(p'-\nabla\cdot\boldsymbol{\Phi})\,\mathbf{U}=0,\quad \mathbf{U}\cdot\boldsymbol{\nu}=0,\quad \nabla\cdot\mathbf{v}=0,$$
$$\mathbf{I}^{\cdot}-2(\mathbf{I}\cdot\boldsymbol{\nu})^s+\mathbf{I}\times\boldsymbol{\omega}-\boldsymbol{\omega}\times\mathbf{I}=0,\quad \mathbf{P}^s=\pi_1\mathbf{U}+\pi_2\mathbf{I}+\pi_3\mathbf{I}\cdot\mathbf{I},\quad w_{si}=\nu_{si}+\mathrm{E}_{sir}\omega^r,$$
$$\omega^{ik}=I^{is}\left(\dot{\omega}_{sk}+\omega_{sr}\omega_{rk}\right),\quad b^k=\mathrm{E}^{ksl}\omega^{sl},\quad b^{(ik)}=\omega^{(ik)},\quad d=\delta_3+2\alpha_7-4\alpha_8,$$

where v, ω, and ν are the translational and rotational velocities and the microdeformation rate; I is the moment of inertia of the particles; U is a unit dyad; ∇ is a space gradient; E^{ksl} is a unit antisymmetric tensor; ρ is the medium density; α_4 is the velocity of rotational friction; α_7 is the viscosity of macro- and microdeformation friction; γ_1, γ_3, γ_{10} and a_{10}, η_{11}, γ_{13} are the viscosities of couple and uncoupled double stresses; δ_3 is microviscosity; α_8, a_{11}, and γ_{11} are the viscosities of the deviatoric-stress, vector, and pseudovector cross-effects; **f** and $\mathbf{B}^S$ are the usual and generalized body forces; π_1, π_2, and π_3 are the constants of the micropressure tensor $\mathbf{P}^S$; $2\mathbf{l}$ is the body moment: parentheses and brackets around the tensor indices denote tensor symmetry and antisymmetry relative to it; the symbols "." and "×" denote scalar and vector multiplication; a superscript dot denotes the total derivative with respect to time; symmetric parts of the dyad are denoted by the superscript "s"; p, p', and Φ are unknown functions arising as a consequence of the macro- and microincompressibility of the liquid; summation is carried out with respect to any index.

The tensors of the force stresses t^{ik}, microstresses s^{ik}, couple stresses $\mathcal{M}^{ik}$, and uncoupled double stresses $\mu^{r(ik)}$ are determined in a Cartesian coordinate system by the equations [7]

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$$
\begin{aligned}
t^{ik} &= -p\delta^{ik} + 2\alpha_7 v_{(i,k)} - 2(\alpha_7 - \alpha_8)\mathrm{v}_{ik};\\
s^{ik} &= -(p + p')\delta^{ik} + 2\alpha_8 v_{(i,k)} + (\delta_3 - 2\alpha_8)\mathrm{v}_{ik} + P_{(ik)};\\
2\mathscr{M}^{pl} &= (\gamma_1 - 1/3\gamma_{10})\,\omega^n{}_{,n}\delta^{pl} + (1/2\gamma_{10} + \gamma_3)\,\omega_{l,p} + (1/2\gamma_{10} - \gamma_3)\,\omega_{p,l} + \gamma_{11}N_{(pl)} + 8a_{11}\mathrm{E}^{plr}N_r;\\
\mu^{r(ik)} &= -\Phi^r\delta^{ik} + \gamma_{13}\mathrm{v}_{(ik,r)'} + 8a_{10}\left(N^{(i}\delta^{k)r} - 1/3N^r\delta^{ik}\right) + 2a_{11}\left(X^{(i}\delta^{k)r} - 1/3X^r\delta^{ik}\right) + 1/4\eta_{11}(\mathrm{E}^{rif}N_{(fk)}\\
&\quad + \mathrm{E}^{rkf}N_{(fi)}) + 1/4\gamma_{11}(\mathrm{E}^{rif}\omega_{(f,k)} + \mathrm{E}^{rkf}\omega_{(f,i)});\\
t^{[ik]} &= \mathrm{E}^{ikl}t^l;\quad t^l = \alpha_4(\mathrm{E}^{lks}v_{s,k} - 2\omega^l);\quad 2N_{kf} = \mathrm{E}^{kqs}\mathrm{v}_{fs,q};\\
4N^l &= 3/2\mathrm{v}^l = \mathrm{v}_{rl,r};\quad \mathrm{v}_{(ik,r)'} = \mathrm{v}_{(ik,r)} - 3/5\mathrm{v}^{(r}\delta^{ik)}.
\end{aligned}
\tag{1.2}
$$

The viscosity coefficients in Eqs. (1.1) and (1.2) satisfy the inequalities

$$
\alpha_4,\ \alpha_7,\ \delta_3,\ \gamma_1,\ \gamma_{10},\ \gamma_3,\ a_{10},\ \eta_{11},\ \delta_3\alpha_7 - 2\alpha_8^2,\ \eta_{11}\gamma_{10} - \gamma_{11}^2,\ a_{10}\gamma_3 - 2a_{11}^2 \geqslant 0. \tag{1.3}
$$

We note that a solution of the system of 20 equations (1.1) for the twenty unknowns $\mathbf{v}$, ω, ν, p, $p' - \nabla\cdot\mathbf{\Phi}$, and $\mathbf{I}$ requires that we have boundary conditions, in addition to the initial conditions. As the translational velocity we take the no-slip conditions $\mathbf{v} = \mathbf{v}_0$, where $\mathbf{v}_0$ is the velocity of the boundary. The boundary conditions for particle rotational velocity and microdeformation rate are written in the form

$$
\begin{aligned}
&2s_4\omega^i - s_2\mathrm{E}^{ijk}v_{k,j} - 2s_8\omega_0^i + 2s_6n^r\mathscr{M}_{ri} = 0,\\
&s_1v_{(i,k)} - s_3\mathrm{v}_{ik} + s_7\overset{0}{\mathrm{v}}_{ik} + s_5n^r\mu^{r(ik)} = 0,
\end{aligned}
\tag{1.4}
$$

where n^r is a normal to the boundary surface.

We note that the conditions (1.4) translate into conditions under which the rotational velocity and microdeformation rate do not cancel out [6] (condition A') when $s_1 = s_2 = s_5 = s_6 = 0$ and $s_3 = s_4 = s_7 = s_8 = 1$. They constitute no-slip conditions (condition A) under the additional constraint, $s_7 = 0$ and under the assumption that ω_0^I coincides with the rotational velocity of the boundary.

These conditions become the Condiff–Dachler condition, and a condition under which the macro- and microdeformation rates are identical [3] (condition B) when $s_1 = s_2 = s_3 = s_4 = 1$, $s_5 = s_6 = s_7 = s_8 = 0$. When $s_4 = s_5 = s_2 = s_8 = s_7 = 0$ and $s_1 = s_3 = s_6 = 1$, Eqs. (1.4) state that couple stresses on the boundary are absent and that the macro- and microdeformation rates are identical (condition C), and, finally, the couple and uncoupled double stresses are absent for $s_5 = s_6 = 1$ and $s_1 = s_2 = s_3 = s_4 = s_7 = 0$. This condition will not be henceforth considered.

We will use the limiting case of the system of equations (1.1), in which the time variation of the moment-of-inertia tensor due to particle rotation and microdeformations will be disregarded in considering the sphere rotation problem. Anisotropy inertia and the presence of a micropressure tensor will not be taken into account. Particular attention will be paid in solving the problem to accounting for the influence of the interaction between microstructure particles of the liquid; the influence of the variation of particle form and orientation in the flow on the solution of the problem will be disregarded.

§2. Within the framework of Eqs. (1.1) we will consider the solution of the problem for a sphere of radius R_0 slowly rotating in an infinite liquid at constant angular rate Ω about the Z axis. The solution of the problem will be found in a spherical coordinate system R, θ, φ. The pressure gradient in the flow field will be set equal to zero. We also assume that the unknowns are independent of time t and coordinate φ. All the physical characteristics will be assumed constant. The $\mathbf{v}$, $\boldsymbol{\omega}$, $\boldsymbol{\nu}$ field velocity will be specified in the form

$$
\begin{gathered}
v_\varphi(R,\ \theta) = v_\varphi(R)\sin\theta,\ \ \omega_r(R,\ \theta) = \omega_r(R)\cos\theta,\\
\omega_\theta(R,\ \theta) = \omega_\theta(R)\sin\theta,\\
\mathrm{v}_{r\varphi}(R,\ \theta) = \mathrm{v}_{r\varphi}(R)\sin\theta,\ \ \omega_\varphi = v_r = v_\theta = \mathrm{v}_{\varphi\varphi} = \mathrm{v}_{\theta\theta} = \mathrm{v}_{\theta\varphi} = 0.
\end{gathered}
\tag{2.1}
$$

Equations (1.1), under the assumptions formulated above and under condition (2.1) in dimensionless form, are written as

$$
\begin{gathered}
\frac{4\mu}{H}\left(\frac{V}{r} - \Omega_r\right) + d_3\frac{df}{dr} + \frac{1}{L_1}\left(D\Omega_r - \frac{2}{r_2}\Omega_r - \frac{4}{r^2}\Omega_\theta\right) - 4\Gamma_3\frac{n}{r} = 0,\\
\frac{2\mu}{H}(BV + 2\Omega_\theta) + d_3\frac{f}{r} - \frac{1}{L_1}\left(D\Omega_\theta - \frac{2}{r^2}\Omega_r\right) - 2\Gamma_3\frac{1}{r^4}\frac{d}{dr}\left(r^4\frac{dN_{r\varphi}}{dr}\right) = 0,\\
DV + 2\mu h - 2HZ_1n = 0,\ CV - \mu_2N_{r\varphi} + \Gamma_3Ch + \frac{1}{L_2}Cn = 0,\ h = B\Omega_\theta + \frac{1}{r}\Omega_r,\\
n = \frac{1}{r^3}\frac{d}{dr}(r^3N_{r\varphi}),\ f = \frac{1}{r^2}\frac{d}{dr}(r^2\Omega_r) + \frac{2}{r}\Omega_\theta,\ \ D = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{2}{r^2},\ B = \frac{d}{dr} + \frac{1}{r},\ C = \frac{d}{dr} - \frac{1}{r}.
\end{gathered}
\tag{2.2}
$$

In obtaining Eqs. (2.2), we introduced the notation

$$VU = v_r,\ \Omega_r U = \omega_r R_0,\ \Omega_0 U = \omega_\theta R_0,\ U = R_0\Omega,\ N_{r\varphi} U = \nu_{r\varphi} R_0,$$
$$rR_0 = R,$$
$$L_{11} = 2L_2(s_{10} + 8/15\mu_9 + 1/16\mu_{66}),\ L_{11} = 2L_1(0{,}5 + s_{310}),$$
$$d_3 = \frac{2}{L_{11}}(s_{110} + 1/6 - s_{310}),$$
$$\Gamma_1 L_{11} = 2s_{11},\ 4\Gamma_2 L_{11} = s_{22},\ \Gamma_3 = \Gamma_1 - \Gamma_2,\ s_{10}\gamma_{10} = a_{10},\ \gamma_1 = s_{110}\gamma_{10},\ \mu_4 = \mu_0 - 2\mu_1^2,$$
$$\mu_9\gamma_{10} = \gamma_{13},\ s_{22}\gamma_{10} = \gamma_{11},\ \mu_{66}\gamma_{10} = \eta_{11},$$
$$s_{11}\gamma_{10} = a_{11},\ s_{310}\gamma_{10} = \gamma_3,\ \mu_2 = 2 + \mu_0 - 4\mu_1,$$
$$\mu_1\alpha_7 = \alpha_8,\ 2R_0^2\alpha_7 = L_{11}\gamma_{10},\ \mu_0\alpha_7 = \delta_3,\ M(\alpha_4 + \alpha_7) = \alpha_4,\ H = 1 - M,\ Z_1 = 1 - \mu_1.$$

The first two equations in (2.2) transform to

$$Df - \lambda_3^2 f = 0,\ 2\mu[L(rV) + 2rh] - \frac{H}{L_1}L(rh) - 2\Gamma_3 L(rn) = 0, \tag{2.3}$$
$$L = \frac{d^2}{dr^2} - \frac{2}{r^2},\ \lambda_3^2 = \frac{4\mu L_1}{H(d_3 L_1 + 1)}.$$

The solution of Eqs. (2.2) and (2.3), which damp at infinity, has the form

$$N_{r\varphi} = \sum_{k=1}^{2} N_k \frac{1}{\sqrt{r}} K_{5/2}(R_k) + \frac{2Z_1}{\mu_2}\frac{N_4}{r^3},\ V = -\sum_{k=1}^{2} N_k \frac{n_k}{\sqrt{r}} K_{3/2}(R_k) - \frac{2}{3}\frac{N_4}{r}, \tag{2.4}$$
$$\Omega_6 = -\sum_{k=1}^{2} N_k \frac{m_k}{R_k}\left[K_{1/2}(R_k) + \frac{1}{R_k}K_{3/2}(R_k)\right]\sqrt{r} - \frac{N_3\sqrt{r}}{R_3^2}K_{3/2}(R_3) - \frac{N_4}{3r^3},$$
$$\Omega_r = -\sum_{k=1}^{2} N_k \frac{2m_k\sqrt{r}}{R_k^2}K_{3/2}(R_k) - \frac{N_3\sqrt{r}}{R_3}\left[K_{1/2}(R_3) + \frac{2}{R_3}K_{3/2}(R_3)\right] - \frac{2}{3}\frac{N_4}{r^3},$$
$$m_k = \frac{\lambda_k^2}{\Delta_k}\left[\frac{\lambda_k^2}{L_2} - \mu_2 + 2HZ_1^2\right],$$
$$n_k = \frac{2}{\Delta_k}\left[HZ_1\lambda_k^2\Gamma_3 + \left(\frac{\lambda_k^2}{L_2} - \mu_2\right)\mu\right],$$
$$\Delta_k = \lambda_k(\Gamma_3\lambda_k^2 - 2\mu Z_1),\ \Gamma_4 = 1 - 2\Gamma_3 L_1 L_2,$$
$$k_3 = \frac{k_0^2}{4} - k_1,\ \Gamma_4 k_1 = 4\mu\mu_4 L_1 L_2,\ \Gamma_4 k_0 = 4\mu L_1 + (2\mu Z_1^2 + \mu_4)L_2 - 8\mu Z_1\Gamma_3 L_1 L_2,$$
$$\lambda_1 = \left(\frac{1}{2}k_0 + k_3^{1/2}\right)^{1/2},\ \lambda_2 = \left(\frac{1}{2}k_0 - k_3^{1/2}\right)^{1/2},$$
$$R_k = \lambda_k r,\ R_3 = \lambda_3 r.$$

Here $K_{n+1/2}$ is a modified half-integral Bessel function. The constants of integration N_1, N_2, N_3, and N_4 are determined from the boundary conditions. For the velocity V we have

$$\sum_{k=1}^{2} n_k N_k K_{3/2}(\lambda_k) + \frac{2}{3}N_4 = -1. \tag{2.5}$$

Conditions (1.4) take the form

$$P_4(1, s_2, s_4) - 2s_8\Omega_r^0 + s_6 P_1(1) = 0,\ \ P_6(1, s_2, s_4) - 2s_8\Omega_\theta^0 + s_6 P_2(1) = 0, \tag{2.6}$$
$$P_5(1, s_1, s_3) - 2s_7\nu_{r\varphi}^0 = 0,\ \alpha_7 UP_2(r)\sin\theta = \mathscr{M}_{r\theta},\ \alpha_7 UP_1(r)\cos\theta = \mathscr{M}_{rr}.$$

Here

$$P_1(r) = \sum_{k=1}^{2} 4N_k t_{k+2}\frac{K_{5/2}(R_k)}{\sqrt{r^3}} + \frac{2N_3}{L_{11}}\left[(s_{110} + 2/3)\frac{K_{3/2}(R_3)}{\sqrt{r}} + \frac{2}{\lambda_3}\frac{K_{5/2}(R_3)}{\sqrt{r^3}}\right] + \frac{4N_4 t_7}{r^4},$$
$$P_2(r) = \sum_{k=1}^{2}\left[2t_{k+2}\frac{K_{5/2}(R_k)}{\sqrt{r_3}} - t_{k+4}\frac{K_{3/2}(R_k)}{\sqrt{r}}\right] + \frac{2N_3}{\lambda_3 L_{11}}\frac{K_{5/2}(R_3)}{\sqrt{r_3}} + \frac{2N_4 t_7}{r^4},$$
$$P_4(r, s_2, s_4) = -\sum_{k=1}^{2} N_k b_k \frac{K_{3/2}(R_k)}{\sqrt{r^3}} + 2s_4 N_3\left[\frac{2K_{3/2}(R_3)}{\lambda_3^2\sqrt{r^3}} + \frac{K_{1/2}(R_3)}{\lambda_3\sqrt{r}}\right] + \frac{4}{3}(s_4 - s_2)\frac{N_4}{r^3},$$

$$P_5(r, s_1, s_3) = \sum_{k=1}^{2} N_k t_k \frac{K_{5/2}(R_k)}{\sqrt{r}} + 2\left(s_1 - 2s_3 \frac{Z_1}{\mu_2}\right)\frac{N_4}{r^3},$$

$$P_6(r, s_2, s_4) = -\sum_{k=1}^{2} N_k b_{k+2}\left[\frac{K_{3/2}(R_k)}{\sqrt{r^3}} + \frac{\lambda_k K_{1/2}(R_k)}{\sqrt{r}}\right] - \frac{2N_3 s_4}{\lambda_3^2}\frac{K_{3/2}(R_3)}{\sqrt{r^3}} - \frac{2}{3}(s_4 - s_2)\frac{N_4}{r^3},$$

$$b_k = 2s_2 n_k - 4s_4 \frac{m_k}{\lambda_k^2},\ b_{k+2} = -\frac{b_k}{2},\ t_{k+2} = \frac{m_k}{\lambda_k L_{11}} + 2\Gamma_2,$$

$$t_{k+4} = 2\Gamma_3\lambda_k - \frac{m_k}{L_1},\ t_7 = \frac{1}{L_{11}} + \frac{4Z_1\Gamma_2}{\mu_2},\ t_k = s_1 n_k \lambda_k - 2s_3.$$

We find the following equations for the components of the force stresses:

$$t_{r\varphi}R_0 = \alpha_7 U \sin\theta\, P_5(r, s_1 = 1, s_3 = Z_1),$$
$$t_{[\theta\varphi]}R_0 = \alpha_7 U \cos\theta P_4\left(r, s_2 = s_4 = \frac{\mu}{H}\right), \qquad (2.7)$$
$$t_{[r\varphi]}R_0 = \alpha_7 U \sin\theta P_6\left(r, s_2 = s_4 = \frac{\mu}{H}\right).$$

The nonzero components of the other stresses will be omitted here as they are cumbersome.

The relative resistance force moments m_c and m_m to the rotation of the sphere, due, correspondingly, to the presence of force stresses and couple stresses, will be calculated using Eq. (2.7) by means of the equations

$$m_c = \frac{M_c}{M_0} = \frac{\alpha_7 U}{M_0} R_0^2 2\pi \int_0^\pi t_{r\varphi}\Big|_{r=1} \sin^2\theta d\theta = 1/3\,[P_6(1) + P_5(1)], \quad M_0 = 8\pi\alpha_7 R_0^3\Omega; \qquad (2.8)$$

$$m_m = \frac{M_m}{M_0} = \frac{\alpha_7 U}{M_0} R_0^2 2\pi \int_0^\pi (M_{rr}\cos\theta - M_{r\theta}\sin\theta)|_{r=1}\sin\theta d\theta = 1/6\,[P_1(1) - 2P_2(1)];$$

$$P_1(1) - 2P_2(1) = \frac{2N_3}{L_{11}}(s_{110} + 2/3)\,K_{3/2}(\lambda_3) + \sum_{k=1}^{2} 2N_k t_{k+4} K_{3/2}(\lambda_k).$$

Here M_0 is torque for ordinary liquid viscosity.

§3. An analysis of the solution (2.4)-(2.9) demonstrates that its nature (flow regime) depends on the type of boundary condition A, A', C, and B and the numerical values of the dimensionless viscosity parameters μ, L_{11}, μ_0, μ_1, μ_{66}, s_{11}, s_{22}, s_{310}, and μ_9. Velocity (V) profiles for possible flow regimes are constructed in Fig. 1 and the velocity profile of a Newtonian liquid is shown by the broken line [10]. Curves for the dependence of the relative moments m_c and m_m and total moment $m = m_c + m_m$ on the parameter L_{11} characterizing slow-scale flow are constructed in Figs. 2a, b, and c.

Curves 1 and 4 in Figs. 1 and 2 were constructed for $\mu = 0.98$ and 0.5, and $L_{11} = 15$, $\mu_0 = 8$, $\mu_1 = 0.6$, $s_{11} = 0$, $s_{22} = 0$, $\mu_9 = 1$, $\mu_{66} = 2$, $s_{10} = 1$, $s_{310} = 1$, and $s_{110} = 2$, respectively. It was assumed for curve 2 that $\mu_1 =$ 1.95. The values $\mu_1 = -0.45$, 0.2, 0.05, −0.15, −1.25, −1.95, −0.25, and −0.05 correspond to curves 3, 5-10, and 12. Curve 11 corresponds to $\mu_0 = 1$. The values of the remaining parameters of curves 2-12 coincide with the parameters of curve 4. We note that curve 3 in Fig. 2 was constructed at one-tenth the scale.

Values corresponding to the solution under the boundary condition A when $\Omega_r^0 = \Omega_\theta^0 = 0$; are depicted by the solid line in Figs. 1 and 2; the solution for condition B is depicted by the curves with points while the solution for conditions C, with circles.

Curves 1 and 4 demonstrate that the velocity V near the rotating sphere in a suspension with deformable particles is greater, under condition A, as in a suspension with solid particles [9], than the speed of an ordinary viscous liquid as a sphere rotates in it. The total resistance force moment to the motion of the sphere in this case is also greater than the classical force moment; it decreases in absolute value with increasing flow scale.

The flow rate of the liquid is less than the classical rate near the sphere (curve 5) in the case of weak interaction between the microstructure of the medium and the surface of the sphere (condition B). Here the suspension particles migrate from the sphere surface or a thin laminar sublayer, observed, for example, in the turbulent motion of a viscous liquid, forms.

The resistance force moment in this case falls with increasing flow scale, unlike the force moment under condition A. These effects are amplified with decreasing viscosity of the deviatoric-stress cross-effect and increasing microviscosity. When $\mu_0 \sim 1$, the velocity is somewhat greater than the classical value and

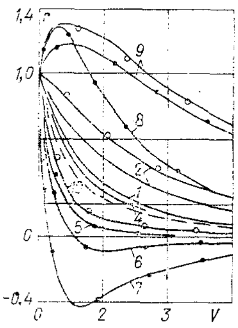

Fig. 1

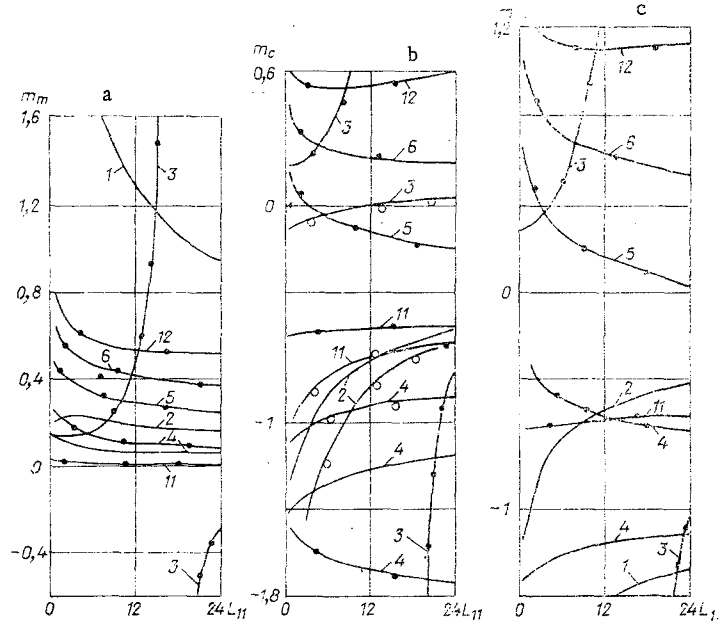

Fig. 2

the resistance force moment less, while it increases with increasing flow scale (curve 11 in Fig. 2b). A similar situation occurs at maximally high values of μ^*_1, at which we have the condition $\mu_4 \geq 0$ implied by the constraints (1.2) (curve 2 in Figs. 1 and 2b, c). Such a solution indicates that the interpretation of the boundary conditions is provisional to some degree and depends on the values of the viscosity parameters of the liquid.

The same features are basically observed for the solution in the case of condition B as in condition C, in which particles may freely rotate near the sphere either in the case of microviscosity on the order of unity or when $\mu_1 = \mu_1^*$.

A decrease in μ_1 under condition B leads to the layer of liquid at a distance from the sphere moving in a direction opposite to its rotation (curves 6 in Fig. 1). This effect is amplified as the negative viscosity μ_1 increases in absolute value to some μ_1^0 (curve 7 in Fig. 1). When μ_1^0 is reached, the nature of the flow sharply varies and pronounced induced flow with velocity profile similar to curve 8, in which the layer of liquid next to the sphere is in advance of its motion, appears. Maximal velocity decreases with a further decrease in μ_1 (curves 8 and 9). Such properties of the solution recall the properties of a flow with a so-called negative viscosity effect [11], in which a difference between the velocities is either maintained or increases if all the other factors permit this [11].

The presence of a point of discontinuity L^*_{11} for the dependence of the frictional force moment on flow scale for a given continuum of negative values of μ_1 can be directly seen from curve 3 (Fig. 2). Consequently, the existence of either a counterflow effect or an induced-flow effect depends on flow scale. A counterflow effect is possible for $L_{11} < L^*_{11}$ if the other flow parameters remain invariant, and induced flow can occur for $L_{11} > L^*_{11}$. Similar effects have also been observed under condition C, the greatest value here corresponding to critical viscosity μ_1^0.

An analysis of the solution under condition A' carried out for values of the viscosity parameters of curve 4 demonstrates that as the boundary parameters increase in absolute value from −15 to 15 (for example, either $\Omega_\theta^0 > 0$ when $\Omega_r^0 = \nu_{r\varphi}^0 = 0$, or $\Omega_r^0 < 0$, when $\Omega_\theta^0 = \nu_{r\varphi}^0 = 0$, or $\nu_{r\varphi}^0 < 0$ when $\Omega_r^0 = \Omega_\theta^0 = 0$) a decrease in flow rate near the sphere that becomes a counterflow effect is observed. Flow rate increases as the parameters Ω_θ^0, Ω_r^0, and $\nu_{r\varphi}^0$ increase, given a change in sign of these parameters, and an induced flow appears as these parameters further increase in the flow. Unlike condition B, no point of discontinuity for the dependence of the resistance force moment on flow scale is observed.

The use of the boundary condition A' leads to the need of introducing additional constants into the theory that require both experimental determination and suitable physical interpretation. This results in definite difficulties. The assumption that viscosity μ_1 is negative, which agrees with the thermodynamic constraints of Eq. (1.3), is optimal in part because turbulent viscosity measured in given flows often turns out to be negative [11].

The effect of negative viscosity for a given flow is described phenomenologically within the framework of a given model (either by assuming a negative viscosity μ_1 or using conditions under which the particle rotational velocity and deformation rate do not cancel out) without making the physical conditions leading to such flows concrete as well as without theoretically determining the values of negative viscosity, which is a characteristic feature of such a description.

We note that a description of the actual mechanism and physical conditions leading to a negative viscosity effects in a number of types of flow can be found in [11], in which the existence of negative turbulence in a system is related to the presence of sources of turbulent kinetic energy, for example, eddies in a system excited from without. From this point of view, the analysis of a solution under the condition that the rotational velocity does not cancel out is of particular interest.

Let us bear in mind that, in contrast to a classical liquid, we have assumed that the viscosities of deformational friction between the macro- and microdeformation motions coincide with the shear viscosity of a classical liquid. It is not necessary to justify such coinciding from a quantitative point of view, since in fact [3] different characteristics of liquids are being equated.

An analysis of the influence of other viscosity parameters on the solution of the problem is omitted here as it is impossible to reduce the great volume of graphical material.

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